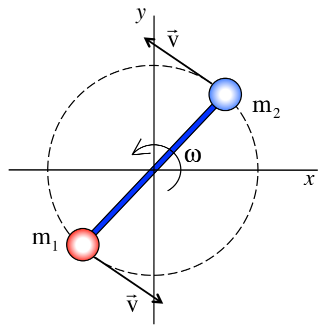
Problem 11.11

What is the *angular momentum* about the system's *center of mass*, as wrapped up in the motion of the system?



There are two relationships that are used to define *angular momentum*. One is the rotational counterpart to standard, translational momentum (i.e., p = mv), but with rotational parameters. That is:

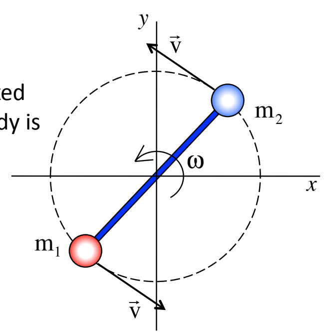
$$L = I\omega$$

The other parallels the force/torque interplay (i.e., $\vec{\Gamma} = \vec{r} \mathrm{x} \vec{F}$) such that it has only translational parameters. That is:

$$\vec{L} = \vec{r} \mathrm{x} \vec{p}$$

We will do this problem using both relationships.

From the purely rotational perspective:



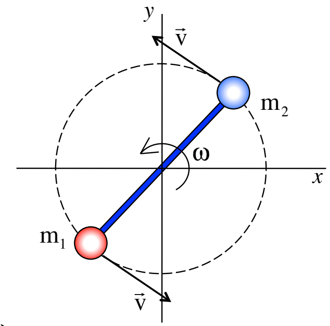
The *moment of inertia* of a point mass about a fixed point is $I = mr^2$. The *angular velocity* of each body is $\omega = v/r$, and r = L/2. As there are two bodies:

$$
\begin{aligned}
L &= I_1\omega + I_2\omega \\
&= \left(m_1 r^{\cancel{2}}\right)\left(\frac{v}{\cancel{r}}\right) + \left(m_2 r^{\cancel{2}}\right)\left(\frac{v}{\cancel{r}}\right) \\
&= \left(m_1 + m_2\right) v \left(\frac{L}{2}\right) \\
&= \left[(4.00 \text{ kg}) + (3.00 \text{ kg})\right](5.00 \text{ m/s})\left(\frac{1.00 \text{ m}}{2}\right) \\
&= 17.5 \text{ kg} \bullet \text{m}^2/\text{s}
\end{aligned}
$$

Because the rotation is *counterclockwise*, the *angular momentum* is positive and the *angular momentum vector* would be expressed as:

$$\vec{L} = \left(17.5 \text{ kg} \bullet \text{m}^2/\text{s}\right)\hat{k}$$

Using the translational parameters:



$$
\begin{aligned}
L &= \vec{r} \mathrm{x} \vec{p}_1 + \vec{r} \mathrm{x} \vec{p}_2 \\
&= |\vec{r}||m_1\vec{v}|\sin\theta + |\vec{r}||m_2\vec{v}|\sin\theta \\
&= \left(\frac{L}{2}\right) m_1|\vec{v}|\sin 90^\circ + \left(\frac{L}{2}\right) m_2|\vec{v}|\sin 90^\circ \\
&= \left(m_1 + m_2\right) v \left(\frac{L}{2}\right) \\
&= \left[(4.00 \text{ kg}) + (3.00 \text{ kg})\right](5.00 \text{ m/s})\left(\frac{1.00 \text{ m}}{2}\right) \\
&= 17.5 \text{ kg} \bullet \text{m}^2/\text{s}
\end{aligned}
$$

You get the same answer either way.